Chapter 16—One-way Analysis of Variance

I am assuming that most people would prefer to see the solutions to these problems as computer printout. (I will use SPSS for consistency.)

16.1 Analysis of Eysenck's data:

a) The analysis of variance:

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              - - - - - O N E W A Y - - - - -
   Variable  RECALL
  By Variable  GROUP      Group Membership
                        Analysis of Variance
                                   Sum of         Mean            F          F
      Source              D.F.       Squares       Squares         Ratio     Prob.
Between Groups              1       266.4500      266.4500      25.2294  .0001
Within Groups              18       190.1000        10.5611
Total                      19       456.5500

                                Standard   Standard
Group       Count       Mean   Deviation      Error      95 Pct Conf Int for Mean

Grp 1         10         19.3000      2.6687       .8439     17.3909  TO  21.2091
Grp 2         10         12.0000      3.7417     1.1832       9.3234  TO  14.6766

Total        20          15.6500      4.9019     1.0961     13.3558  TO  17.9442
```

b) *t* test

```
t-tests for Independent Samples of GROUP    Group Membership
                         Number
 Variable              of Cases       Mean          SD   SE of Mean
 ----------------------------------------------------------------------
 RECALL
 Young                     10       19.3000       2.669      .844
 Older                     10       12.0000       3.742     1.183
 ----------------------------------------------------------------------
          Mean Difference = 7.3000
         Levene's Test for Equality of Variances: F= .383   P= .544

       t-test for Equality of Means                              95%
 Variances    t-value    df    2-Tail Sig     SE of Diff    CI for Diff
 ----------------------------------------------------------------------
 Equal        5.02       18        .000        1.453    (4.247, 10.353)
 Unequal      5.02       16.27     .000        1.453    (4.223, 10.377)
```

Notice that if you square the *t* value of 5.02 you obtain 25.20, which is the same as the *F* in the analysis of variance. Notice also that the analysis of variance procedure produces confidence limits on the means, whereas the *t* procedure produces confidence limits on the difference of means.

16.3 Expanding on Exercise 16.2:

a) Combine the Low groups together and the High groups together:

```
      Variable  RECALL
   By Variable  LOWHIGH
                                 Analysis of Variance
                             Sum of        Mean           F      F
        Source        D.F.   Squares       Squares        Ratio  Prob.
Between Groups          1    792.1000     792.1000      59.4505  .0000
Within Groups          38    506.3000      13.3237
Total                  39   1298.4000

                       Standard   Standard
Group   Count    Mean   Deviation      Error    95 Pct Conf Int for
Mean

Grp 1    20    6.7500    1.6182      .3618     5.9927  TO    7.5073
Grp 2    20   15.6500    4.9019    1.0961    13.3558  TO   17.9442

Total    40   11.2000    5.7699      .9123     9.3547  TO   13.0453
```

Here we have compared recall under conditions of Low versus High processing, and can conclude that higher levels of processing lead to significantly better recall.

b) The answer is still a bit difficult to interpret because both groups contain both younger and older subjects, and it is possible that the effect holds for one age group but not for the other.

16.5 η^2 and ω^2 for the data in Exercise 16.1:

$SS_{group} = 266.45$
$SS_{total} = 456.55$
$MS_{error} = 10.564$
$k = 2$

$$\eta^2 = \frac{SS_{group}}{SS_{total}} = \frac{266.45}{456.55} = .58$$

$$\omega^2 = \frac{SS_{group} - (k-1)MS_{error}}{SS_{total} + MS_{error}}$$

$$= \frac{266.45 - (2-1)10.564}{456.55 + 10.564} = \frac{255.886}{467.114} = .55$$

16.7 Foa *et al*. (1991) study:

Group	***n***	**Mean**	**S.D.**	**Total**	**Variance**
SIT	14	11.07	3.95	155	15.6025
PE	10	15.40	11.12	154	123.6544
SC	11	18.09	7.13	199	50.8369
WL	10	19.50	7.11	195	50.5521
Total	45	15.622		703	

$$\bar{X}.. = \frac{703}{45} = 15.622$$

$$SS_{treat} = \sum n_j \left(\bar{X}_j - \bar{X}..\right)^2$$

$$= 14(11.07-15.622)^2 + 10(15.40-15.622)^2 + 11(18.09-15.622)^2 + 10(19.50-15.622)^2$$

$$= 507.840$$

$$MS_{error} = \frac{\sum (n_i - 1)s_i^2}{\sum (n_i - 1)}$$

$$= \frac{13(15.6025) + 9(123.6544) + 10(50.8369) + 9(50.5521)}{41}$$

$$= 55.587$$

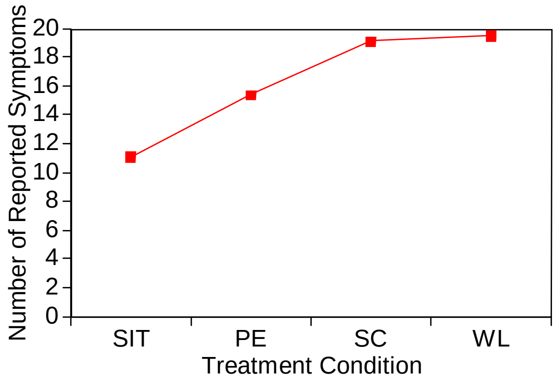
$$SS_{error} = [\Sigma(n_1 - 1)]MS_{error} = 41 * 55.587 = 2279.067$$

From these values we can fill in the complete summary table and compute the *F* value.

Source	***df***	***SS***	***MS***	***F***
Treatment	3	507.840	169.280	3.04
Error	41	2279.067	55.587	
Total	44	2786.907		

[$F_{.05}(3,41) = 2.84$] We can reject the null hypothesis and conclude that there are significant differences between groups. Some treatments are more effective than others.

b)



c) It would appear that the more interventionist treatments lead to fewer symptoms than the less interventionist ones, although we would have to run multiple comparisons to tell exactly which groups are different from which other groups.

16.9 If the sample sizes in Exercise 16.7 were twice as large, that would double the SS_{treat} and MS_{treat}. However it would have no effect on MS_{error}, which is simply the average of the group variances. The result would be that the *F* value would be doubled.

16.11 Effect size for tests in Exercise 16.10.
It only makes sense to calculate an effect size for significant comparisons in this study, so we will deal with SIT vs SC.

$$\hat{d} = \frac{\bar{X}_{SC} - \bar{X}_{SIT}}{\sqrt{MS_{error}}} = \frac{18.09 - 11.07}{\sqrt{55.579}} = \frac{7.02}{7.455} = 0.94$$

The SIT group is nearly a full standard deviation lower in symptoms when compared to the SC group, which is a control group.

16.13 ANOVA on GPAs for the ADDSC data:

Variable GPA
By Variable Group

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	22.5004	11.2502	22.7362	.0000
Within Groups	85	42.0591	.4948		
Total	87	64.5595			

Group	Count	Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean
Grp 1	14	3.2536	.5209	.1392	2.9528 TO 3.5543
Grp 2	49	2.5920	.6936	.0991	2.3928 TO 2.7913
Grp 3	25	1.7436	.8020	.1604	1.4125 TO 2.0747
Total	88	2.4563	.8614	.0918	2.2737 TO 2.6388

There is a significant difference between the groups, telling us that there is a relationship between ADDSC score in elementary school and the GPA the student has in 9th grade. From the means it is clear that the GPA declines as the ADDSC score increases.

These are real data, and they tell us that a teacher in elementary school can already pick out those students who will do well and badly in high school. I have always found these results depressing and worrisome, even though psychologists are supposed to like to be able to predict. There are some things I wish weren't so predictable.

16.15 Analysis of Darley and Latané data:

Group	***n***	**Mean**	**Total**
1	13	0.87	11.31
2	26	0.72	18.72
3	13	0.51	6.63
Total	52		36.66

$$\begin{aligned} SS_{treat} &= \Sigma n_j\left(\bar{X}_j - \bar{X}..\right)^2 \\ &= 13(0.87-0.705)^2 + 26(0.72-0.705)^2 + 13(0.51-0.705)^2 \\ &= 0.8541 \end{aligned}$$

$$MS_{error} = 0.053 \quad \text{(given in text)}$$

$$SS_{error} = [\Sigma(n_1 - 1)]MS_{error} = 49 * 0.053 = 2.597$$

From these values we can fill in the complete summary table and compute the *F* value.

Source	***df***	***SS***	***MS***	***F***
Treatment	2	0.854	0.427	8.06
Error	49	2.597	0.053	
Total	51	3.451		

[$F_{.05}(2,49) = 3.18$] We can reject the null hypothesis and conclude that subjects are less likely to summon help quickly if there are other bystanders around.

16.17 Bonferroni test on data in Exercise 16.2:

Both of these comparisons will be made using *t* tests. The means are given in Exercise 16.15 above.

$$t = \frac{\overline{X}_i - \overline{X}_j}{\sqrt{\frac{MS_{error}}{n_i} + \frac{MS_{error}}{n_j}}}$$

For Young/Low versus Old/Low:

$$t = \frac{6.5 - 7.0}{\sqrt{\frac{6.6278}{10} + \frac{6.6278}{10}}} = \frac{-0.5}{1.151} = -0.434$$

For Young/High versus Old/High:

$$t = \frac{19.3 - 12.0}{\sqrt{\frac{6.6278}{10} + \frac{6.6278}{10}}} = \frac{7.3}{1.151} = 6.34$$

For 36 *df* for error and for 2 comparisons at a familywise error rate of $\alpha = .05$, the critical value of $t = 2.34$. There is clearly not a significant difference between young and old subjects on tasks requiring little cognitive processing, but there is a significant difference for tasks requiring substantial cognitive processing. The probability that *at least* one of these statements represents a Type I error is at most .05.

It is worth pointing out that when we are using MS_{error} as our variance estimate, and have equal sample sizes, the computations are very simple because we only need to calculate the denominator once.

16.19 Effect size for WL versus SIT

$$\hat{d} = \frac{\overline{X}_{WL} - \overline{X}_{SIT}}{s_{WL}} = \frac{19.50 - 11.07}{7.11} = \frac{8.43}{7.11} = 1.18$$

The two groups differ by over a standard deviation.

16.21 Spilich *et al*. data on a cognitive task:

Variable ERRORS
By Variable SMOKEGRP

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	2643.3778	1321.6889	4.7444	.0139
Within Groups	42	11700.4000	278.5810		
Total	44	14343.7778			

Group	Count	Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean
Grp 1	15	28.8667	14.6866	3.7921	20.7335 TO 36.9998
Grp 2	15	39.9333	20.1334	5.1984	28.7838 TO 51.0828
Grp 3	15	47.5333	14.6525	3.7833	39.4191 TO 55.6476
Total	45	38.7778	18.0553	2.6915	33.3534 TO 44.2022

Here we have a task that involves more cognitive involvement, and it does show a difference due to smoking condition. The non-smokers performed with fewer errors than the other two groups, although we will need to wait until the next exercise to see the multiple comparisons.

16.23 Spilich *et al*. data on driving simulation:

Variable ERRORS
By Variable SMOKEGRP

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	437.6444	218.8222	9.2584	.0005
Within Groups	42	992.6667	23.6349		
Total	44	1430.3111			

Group	Count	Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean
Grp 1	15	2.3333	2.2887	.5909	1.0659 TO 3.6008
Grp 2	15	6.8000	5.4406	1.4048	3.7871 TO 9.8129
Grp 3	15	9.9333	6.0056	1.5506	6.6076 TO 13.2591
Total	45	6.3556	5.7015	.8499	4.6426 TO 8.0685

Here we have a case in which the active smokers again performed worse than the non-smokers, and the differences are significant.

16.25 Attractiveness of faces

a) The research hypothesis would be the hypothesis that faces averaged over more photographs would be judged more attractive than faces averaged over fewer photographs.

b) Data analysis

Descriptives

ATTRACT

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
1.00	6	2.60467	.431353	.176099	2.15199	3.05734	2.201	3.380
2.00	6	2.64500	.657059	.268243	1.95546	3.33454	1.893	3.644
3.00	6	2.89000	.447100	.182528	2.42080	3.35920	2.118	3.422
4.00	6	3.18500	.208053	.084937	2.96666	3.40334	2.860	3.505
5.00	6	3.26000	.068118	.027809	3.18852	3.33148	3.169	3.357
Total	30	2.91693	.473378	.086427	2.74017	3.09370	1.893	3.644

ANOVA

ATTRACT

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	2.170	4	.543	3.134	.032
Within Groups	4.328	25	.173		
Total	6.499	29			

c) Conclusions

The group means are significantly different. From the descriptive statistics we can see that the means consistently rise as we increase the number of faces over which the composite was created.